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The Weak Mixing Angle From TeV Scale Quark-Lepton Unification

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Abstract

Unification of quarks and leptons via left-right symmetry, $SU(4) \times SU(2)_L \times SU(2)_R$, is extended to $SU(4) \times SU(2)^4$. All four $SU(2)$ interactions have the same gauge coupling, leading to a tree-level prediction of the weak mixing angle, $\sin^2\theta = 0.239$, which is so close to data that this unification must occur around the TeV scale. Five dimensional theories are constructed with boundary condition breaking of $SU(4) \to SU(3) \times U(1)_{B-L}$, removing powerful constraints from $K_L \to \mu e$ while allowing a reliable calculation of the leading logarithm corrections to $\sin^2\theta$. The compactification scale is expected in the 1–5 TeV region, depending on how $SU(2)^4$ is broken. Two illustrative models are presented, and the experimental signal of the Z' gauge boson is discussed.

1 Introduction

The strong, weak and electromagnetic forces, so differently manifested in nature, are described by underlying gauge interactions based on common principles. From this similarity in structure it becomes highly plausible that these three interactions are just low energy remnants of a more unified gauge theory. Such a unified theory could provide predictions for ratios of gauge couplings and ratios of fermion masses, as well as an understanding of the quark and lepton quantum numbers.

The most basic question about such a unification of the gauge forces is the mass scale at which the unification occurs. The standard paradigm of unification into a supersymmetric $SU(5)$ or $SO(10)$ theory [1] has a high unification scale, of order 10^{16} GeV, and leads to the highly successful prediction for the weak mixing angle of $\sin^2\theta = 0.233 \pm 0.002$. Objections to the simplest model, about proton decay, the lightness of the Higgs doublets compared to their color triplet partners and quark-lepton mass relations, can all be overcome by promoting $SU(5)$ to a five-dimensional (5D) gauge symmetry broken by boundary conditions in a 5th dimension, which simultaneously improves the prediction of the weak mixing angle to $\sin^2\theta = 0.2313 \pm 0.0004$ [2]. Although the unification mass scale is reduced to near 10^{15} GeV, it is still extremely large.

It has recently been argued that a successful prediction of the weak mixing angle is also possible if the unification occurs at a low scale, in the TeV domain [3]. It is well known that the $SU(2) \times U(1)$ electroweak theory can be embedded into $SU(3)_{EW}$ in a straightforward way for leptons [4]. In this scheme, hypercharge is identified as the diagonal generator of $SU(3)$ which is orthogonal to $SU(2)_L$:

$$\frac{Y}{2} = \sqrt{3}\, T_8, \tag{1}$$

leading to the tree level prediction for the weak mixing angle, $\sin^2\theta = 0.25$. This prediction is intriguingly close to the experimental value of $\sin^2\theta$ at the Z pole [5],

$$\sin^2\theta(M_Z)|_{exp} = 0.23113 \pm 0.00015. \tag{2}$$

Refining the tree-level prediction of the $SU(3)_{EW}$ model by including one-loop radiative corrections leads to a correct prediction of $\sin^2\theta(M_Z)$ provided that the unification scale (the scale at which $SU(3)_{EW}$ breaks down to $SU(2) \times U(1)$) is about 4 TeV. The main difficulty of this unification scheme is that the quarks do not exhibit any $SU(3)$ pattern. This problem can be overcome if $SU(2)_L \times U(1)_Y$ is embedded into $SU(2) \times U(1) \times SU(3)$ in such a way that the $SU(3)$ factor is most important in determining the value of the low energy couplings [3]. Alternatively one can construct an $SU(3)_{EW}$ theory in 5D, with boundary condition breaking to $SU(2)_L \times U(1)_Y$ [6, 7, 8]. The quarks live on the boundary where only the $SU(2)_L \times U(1)_Y$ gauge symmetry is operative, while the leptons and Higgs may feel the full $SU(3)_{EW}$ gauge symmetry[1].

[1] For subsequent work along the lines of $SU(3)_{EW}$ unification see, for example, Refs. [9, 10].

The $SU(3)_{EW}$ model of TeV scale unification can be criticized on the grounds that the prediction for the weak mixing angle is less precise than in the case of high scale unification. Also, this unification scheme gives no understanding of the quantum numbers of the quarks[2]. Nevertheless, it provides a testable alternative to the high-scale unification, and becomes especially interesting if the hierarchy problem is resolved by lowering the fundamental gravity scale to the TeV region [12] or by introducing a warped extra dimension [13].

In this paper we introduce a new idea for TeV scale unification. Our starting point is the model of Pati and Salam [14] based on the $SU(4) \times SU(2)_L \times SU(2)_R$ gauge group. This model provides a very satisfactory understanding of both quark and lepton quantum numbers. Moreover, the additional gauge bosons of this model do not lead to proton decay, and the constraints on the unification scale are much weaker than for the $SU(5)$ or $SO(10)$ gauge groups. By itself, however, the Pati-Salam model does not provide a viable scheme for TeV scale unification. In this model, hypercharge is obtained as a linear combination of the $B-L$ generator of $SU(4)$ and the diagonal generator of $SU(2)_R$, leading to a grossly incorrect tree-level prediction for $\sin^2\theta$:

$$Y = T_{B-L} + T_{3R} \qquad \Longrightarrow \qquad \sin^2\theta = \frac{1}{2} - \frac{1}{6}\,\frac{\alpha}{\alpha_s}\Big|_{M_Z} = 0.489, \tag{3}$$

where we have used the values of the Standard Model (SM) gauge couplings at M_Z[3]. (We have imposed an additional Z_2 symmetry interchanging the $SU(2)$ factors; without such a symmetry, the model does not make a definite prediction for $\sin^2\theta$.) Clearly, radiative corrections cannot render the prediction (3) consistent with the experimental result (2) if the unification scale is in the TeV range. To rectify this problem, we propose to enlarge the gauge group of the Pati-Salam model to include two additional $SU(2)$ factors, which we will refer to as $SU(2)_1$ and $SU(2)_2$. The Standard Model quarks and leptons are not charged under the additional $SU(2)$'s; as we discuss below, the model may or may not contain additional matter charged under these groups. We assume a discrete symmetry that interchanges the four $SU(2)$ factors, making their (ultraviolet) gauge couplings identical. Crucially, the hypercharge arises as a linear combination of the $B-L$ generator of $SU(4)$ and the T_3 generator of the *diagonal subgroup* of the $SU(2)_R \times SU(2)_1 \times SU(2)_2$. This embedding of the hypercharge leads to the tree-level prediction for the weak mixing angle which is remarkably close to experiment:

$$Y = T_{B-L} + T_{3R} + \sum_{i=1}^{2} T_{3,i} \qquad \Longrightarrow \qquad \sin^2\theta = \frac{1}{4} - \frac{1}{6}\,\frac{\alpha}{\alpha_s}\Big|_{M_Z} = 0.239. \tag{4}$$

Below, we will investigate the radiative corrections to this formula, and present explicit models of TeV-scale unification which lead to a prediction of $\sin^2\theta(M_Z)$ consistent with (2).

[2]The four-dimensional version of the model [3] is also subject to severe electroweak precision constraints [11].

[3]In deriving Eq. (3), we have treated the $SU(2)$ and $SU(4)$ couplings as independent, and used their experimental values at M_Z as inputs. An alternative is to assume that the Pati-Salam group is embedded in $SO(10)$. In this case, the model predicts $\sin^2\theta = 3/8$ at the unification scale, which is also unacceptable for TeV scale unification.

2 Boundary Condition Breaking of $SU(4)$

There are three immediate objections to realising the idea outlined above:

- As the tree-level prediction for the mixing angle, Eq. (4), is only 3% away from its experimental value at the Z pole, the $4-2^4$ structure has to be realized at a scale not too much higher than M_Z, at most around a TeV. (Otherwise, radiative corrections are too large and destroy the successful prediction of $\sin^2\theta(M_Z)$.) This requirement seems to contradict the experimental lower bounds of about 1000 TeV on the mass of the exotic $SU(4)/(SU(3)\times U(1)_{B-L})$ gauge bosons X. (A strong constraint arises from the non-observation of $K_L \to \mu e$ [15].)
- A Standard Model generation consists of two $SU(4)$ 4-plets: ψ_L, which is a doublet under $SU(2)_L$, and ψ_R, which is a doublet under $SU(2)_R$. There are no fields charged under the additional two $SU(2)$'s. Hence, the simplest interpretation of a generation does not allow a discrete symmetry which ensures equality of the four $SU(2)$ gauge couplings, as needed for a prediction of the weak mixing angle.
- The $SU(4)$ symmetry constrains the up quarks and neutrinos to have Yukawa couplings of the same size. Even if the right handed neutrinos receive Majorana masses at the TeV scale, the three light neutrinos all have masses far in excess of experimental limits.

Although there may be several answers to these objections, in this paper we pursue just a single idea. We assume that the symmetry breaking $SU(4)\to SU(3)\times U(1)_{B-L}$ is accomplished by boundary conditions in a compact extra dimension x_5 of physical length πR, with $R\sim \text{TeV}^{-1}$. Explicitly, we start with a 5D $SU(4)$ gauge field $A_M \equiv A_M^a T^a, M=1\dots5, a=1\dots15$, and impose the following boundary conditions:

$$\begin{aligned}A_\mu(x^\mu,x_5) &= +A_\mu(x^\mu,-x_5) = Z\,A_\mu(x^\mu,x_5+2\pi R)\,Z^{-1},\\ A_5(x^\mu,x_5) &= -A_5(x^\mu,-x_5) = Z\,A_5(x^\mu,x_5+2\pi R)\,Z^{-1},\end{aligned} \tag{5}$$

where $\mu=1\dots4$ and $Z=$diag$(+,+,+,-)$. The low-energy effective field theory contains nine four-dimensional (4D) massless gauge bosons A_μ of the $SU(3)\times U(1)_{B-L}$ group, while the remaining six gauge bosons X_μ and the fifteen 4D scalars A_5^a do not possess zero modes. We assume that the gauge bosons of the four $SU(2)$ groups of our model are free to propagate in the bulk; at this point, we leave open the question as to whether $SU(2)_R\times SU(2)_1\times SU(2)_2$ is broken by boundary conditions or by Higgs vevs. These assumptions have the virtue of overcoming all three objections in an economical way:

- The constraints on the X boson mass are naturally avoided if the matter fields live in the bulk. Consider a 5D 4-plet $\Psi=(Q,L)$. Due to the boundary condition breaking of $SU(4)$, only one component of Ψ has a zero mode. (This component can be either L or Q, depending on the charge of Ψ under the reflection.) Thus, one generation of SM fermions requires *four* 5D 4-plets: $\Psi_L = (Q_L, \tilde{L}_L)$, $\Psi'_L = (\tilde{Q}_L, L_L)$, $\Psi_R = (Q_R, \tilde{L}_R)$, and $\Psi'_R = (\tilde{Q}_R, L_R)$, where the tildes mark the fields that do not possess zero modes. The SM quarks and leptons of the same generation *do not* come from the same $SU(4)$ multiplet, and are not coupled through the X bosons of $SU(4)$. (This lack of unification does not destroy the understanding of the quantum numbers of a generation provided by $4-2-2$!)

Alternatively, SM generations can be built out of four-dimensional fields living on a boundary of space-time where the $SU(4)$ symmetry is broken to $SU(3)\times U(1)$ (the "3-1 point"). The X boson wave function vanishes at this boundary. Note, however, that in this case the $4-2-2$ pattern of the quark and lepton quantum numbers is purely accidental.

- If quarks and leptons live in the bulk, a discrete symmetry relating the four $SU(2)$ factors can be imposed provided that we introduce three additional generations of fermions. Each additional generation consists of four $SU(4)$ 4-plets, two of them transforming as doublets under $SU(2)_1$ and the other two under $SU(2)_2$. The extra fermions acquire masses at the $SU(2)_1 \times SU(2)_2$ breaking scale, and are therefore sufficiently heavy to escape detection.

  If quarks and leptons live on the 3-1 point, it is not necessary to introduce new fields. In this case, a discrete symmetry ensuring the equality of the four $SU(2)$ couplings can be imposed in the bulk. This symmetry has to be broken at the boundary, but the corrections to the weak mixing angle prediction due to this breaking are suppressed by the volume of the fifth dimension. As we explain below, this volume (in units of the fundamental scale) is taken to be large in our model, making these corrections irrelevant.

- We take the Yukawa couplings to be located on the 3-1 boundary. If the fermions live on this boundary, this is automatic. If the fermions live in the bulk, the absence of bulk Yukawa couplings could be due to supersymmetry, or to the fact that the Higgs field is localized on the boundary. Since only the $SU(3) \times U(1)_{B-L}$ subgroup of the $SU(4)$ gauge interactions are operative at the 3-1 point, there is no relation between the Yukawa couplings of the up quarks and the neutrinos.

Promoting the gauge symmetry of our model to 5D raises an important issue. Since a 5D gauge theory is non-renormalizable, there has to be a fundamental scale M_s at which it breaks down. As we will see below, in our model $M_s \sim 100$ TeV. While proton stability at the renormalizable level is ensured by the accidental symmetries of the Pati-Salam model, higher-dimension operators induced at M_s could lead to proton decay. To prevent that, we will impose global B and L symmetries. Since quarks and leptons come from different $SU(4)$ multiplets, these symmetries commute with the gauge transformations. There is also a possibility of additional flavor violating effects (e.g. $K_L \to \mu e$ decays) induced by the non-renormalizable operators generated at M_s. However, since the scale is rather high, even a very modest amount of flavor symmetry in the fundamental theory (or small fine tuning) would be sufficient to render these effects unobservable at present.

3 Radiative Corrections and Uncertainties in the Prediction of the Weak Mixing Angle

Does the tree-level prediction (4) survive in the 5D theory with boundary condition breaking of $SU(4)$? At tree level, the 4D gauge couplings are given by

$$\frac{1}{g_{i,4}^2} = \frac{\pi R}{g_{i,5}^2} + \frac{1}{\tilde{g}_i{}^2}, \tag{6}$$

where $g_{i,5}$ is the corresponding 5D gauge coupling, and $1/\tilde{g}_i{}^2$ is the coefficient of the gauge kinetic term induced on the $3-1$ boundary. The equality of the four $SU(2)$ couplings and the equality of the strong coupling and the appropriately normalized $U(1)_{B-L}$ coupling, which were crucial in obtaining the result (4), relied on the symmetries which are realized in the bulk but not at the $3-1$ boundary. Therefore, they hold for the terms involving $g_{i,5}$ but not for the boundary-induced terms proportional to $1/\tilde{g}_i{}^2$. The boundary-induced terms could therefore lead to large corrections to (4). Such effects were also present in the 5D $SU(5)$ [2] and the 5D $SU(3)_{EW}$ [7] unification schemes, and we follow the assumptions made there to recover a high degree of predictivity: we take the 5D theory as the correct effective theory up to the scale M_s at which all bulk and boundary gauge interactions are assumed to become strong. The strong bulk coupling assumption implies the existence of a hierarchy between M_s and the compactification scale $M_c = 1/R$: $M_s/M_c \approx l_5/(\pi C_2 g^2) \approx 32\pi^2/(C_2 g^2) \gg 1$, where C_2 and g are the quadratic Casimir and the low-energy gauge coupling, respectively, and $l_5 = 32\pi^3$ is the 5D loop factor [16]. The precise magnitude of the hierarchy is rather uncertain due to an approximate nature of the strong coupling argument. In our numerical estimates, we will use $M_s/M_c = 80$; this estimate is probably valid to about a factor of two. According to (6), this means that the boundary gauge kinetic terms are subdominant to the bulk ones.

The leading logarithmic corrections to the formula (4) can be readily calculated. We will make a simplifying assumption that the gauge symmetry breaking of $SU(2)_R \times SU(2)_1 \times SU(2)_2 \to T_{3R}$ involves no mass scale other than M_s and M_c. With these assumptions, the prediction for the weak mixing angle at leading logarithm is

$$\begin{aligned}\sin^2\theta(M_Z) = \frac{1}{4} - \frac{1}{6}\,\frac{\alpha}{\alpha_s}\Big|_{M_Z} &- \frac{\alpha(M_Z)}{8\pi}\left(b_Y - 3b_2 - \frac{2}{3}b_3\right)\ln\frac{M_c'}{M_Z} \\ &- \frac{\alpha(M_Z)}{8\pi}\left((b_{B-L}' - \frac{2}{3}b_3') + (b_R' - 3b_2')\right)\ln\frac{M_s}{M_c'},\end{aligned} \tag{7}$$

where $b_{3,2,Y}$ are the beta function coefficients for $g_{3,2,Y}$ below the modified compactification scale $M_c' = M_c/\pi$ [2, 7], while $b'_{3,2,B-L}$ are the beta function coefficients for the relative logarithmic running of the QCD, $SU(2)_L$, and $B-L$ gauge couplings above M_c'. The formula (7) is sufficiently general to be used for various patterns of the $SU(2)_R \times SU(2)_1 \times SU(2)_2 \to T_{3R}$ symmetry breaking: this model dependence is encoded in the coefficient b'_R. For example, if the breaking occurs entirely by the Higgs mechanism at M_c, then b'_R is given by the sum of the beta function coefficients describing the relative logarithmic running of the $SU(2)_R$, $SU(2)_1$, and $SU(2)_2$ gauge couplings above M_c'; if the breaking occurs entirely by the Higgs mechanism at scale M_s, b'_R is just equal to the beta function coefficient for the evolution of the single gauge coupling $g_{T_{3R}}$; etc.

Above the compactification scale, the gauge couplings run according to a power law; however, the leading corrections to the $\sin^2\theta$ prediction are logarithmic. This is due to *universality* of the power-law running: since the discrete symmetry relating the four $SU(2)$ factors of our model is broken only locally in the 5D bulk, the four gauge couplings have identical power law running. By the same argument, the boundary condition breaking of $SU(4)$ ensures that the power running of the $SU(3)$ and the (appropriately normalized) $U(1)_{B-L}$ coupling constants is identical, and all the power running effects cancel out in Eq. (7).

If the theory below M_c is the Standard Model, the running between this scale and M_Z corrects the weak mixing angle by $\delta \sin^2\theta = -0.0065 \ln(M_c'/M_Z)$. The rules for computing the beta function coefficients above the compactification scale in orbifold models were given in [2]. Applying these rules to running of the $SU(3)$ and $U(1)_{B-L}$ gauge couplings gives $b'_{B-L} - (2/3)b'_3 = 23/6$. Since matter fields come in complete $SU(4)$ multiplets, they do not contribute to this combination of beta functions; therefore, this result does not depend on whether the matter fields live in the bulk or on the boundary. It is also clearly independent of the pattern of the $SU(2)_R \times SU(2)_1 \times SU(2)_2 \to T_{3R}$ breaking. Inserting these results into (7), we obtain

$$\sin^2\theta(M_Z) = 0.234 - 0.0065 \ln(M_c'/M_Z) - 0.0015\,(b'_R - 3b'_2) \pm 0.005. \tag{8}$$

The $b'_R - 3b'_2$ term in this equation is the model-dependent contribution from relative running of the $SU(2)$ gauge couplings. We will evaluate it in some explicit models in the next section. Before doing that, however, let us make the following important observation. The second term of Eq. (8) is negative. If the third term is non-positive, $b'_R - 3b'_2 \geq 0$, reproducing the correct value of the weak mixing angle requires a very low value of M_c, inconsistent with experimental constraints. Thus, the successful models will be those with $b'_R - 3b'_2$ negative, tending to increase M_c.

It is also interesting to consider the supersymmetric version of our model, with the supersymmetry breaking scale in the visible sector $\tilde{m} \lesssim M_c$. In this case, we obtain $b'_{B-L} - (2/3)b'_3 = 4$, and Eq. (7) becomes

$$\sin^2\theta(M_Z) = 0.234 - 0.0031 \ln(M_c'/M_Z) - 0.0015\,(b'_R - 3b'_2) \pm 0.005, \tag{9}$$

where we have not included supersymmetric threshold corrections involving $\ln(\tilde{m}/M_Z)$. Note that the coefficient of the second term in this equation is smaller than in its non-supersymmetric counterpart, implying that it is easier to construct realistic models in the supersymmetric case.

The uncertainties in the weak mixing angle predictions (8) and (9) come from two sources. First, as we already explained, the predictions can be corrected at tree level by the gauge kinetic terms induced on the 3-1 boundary. The dominant effect comes from the violation of the discrete symmetry relating the four $SU(2)$ gauge couplings on that boundary. Using Eq. (6) and the strong coupling assumption, we estimate that the boundary terms correct each of the low energy gauge couplings by an amount $\delta_i \equiv \delta g_i^2/g_i^2 \approx l_4/(l_5 M_s \pi R) \approx 1/40$, where $l_4 = 16\pi^2$ is the 4D loop factor [16]. The uncertainty in the tree-level prediction for the weak mixing angle is given by $\delta \sin^2\theta/\sin^2\theta \approx 0.25 \sum_i \delta_i$; assuming that the uncertainties from the four $SU(2)$'s add in quadruture, we arrive at an estimate $\delta \sin^2\theta \approx 0.003$. The second source of uncertainty is the non-logarithmically enhanced loop contributions from running between M_s and M_c'; since $\ln(M_s/M_c') \approx 5$, we estimate this uncertainty to be 20% of the term in Eq. (7) involving this logarithm. Numerically, this corresponds to $\delta \sin^2\theta \approx 0.002$, slightly below 1%. Thus, we estimate the total uncertainty in our prediction of $\sin^2\theta(M_Z)$ to be about 2%, or 0.005. Note that this theoretical uncertainty can be reduced in specific models. For example, if the discrete symmetry relating the four $SU(2)$ gauge couplings is only broken *spontaneously* on the 3-1 boundary, it will be respected by the boundary gauge kinetic terms, and they will not modify the weak mixing angle prediction. However, we will not pursue this possibility in this paper.

4 Sample Models

In this section, we present two explicit models realizing the ideas discussed above which lead to acceptable predictions for the weak mixing angle. In both models, the $SU(4) \to SU(3) \times U(1)_{B-L}$ breaking is achieved by imposing boundary conditions as described in section 2. Both models are supersymmetric, with supersymmetry broken spontaneously at scales of order TeV. A 5D gauge supermultiplet consists of a symplectic Majorana spinor λ_i and a real scalar σ, in addition to the 5D vector field A_M. The boundary conditions for the fields in the $SU(4)$ gauge multiplet are given by the equation (5), with $\lambda_{1+} = \frac{1}{2}(1+\gamma_5)\lambda_1$ transforming like A_μ while σ and $\lambda_{2+} = \frac{1}{2}(1+\gamma_5)\lambda_2$ transform like A_5. The zero modes of A_μ and λ_{1+} form a 4D $N=1$ gauge multiplet, while the other fields have no zero modes.

The remaining freedom concerns the location of the matter fields, the mechanism of breaking $SU(2)_R \times SU(2)_1 \times SU(2)_2 \to T_{3R}$, and the structure of the Higgs sector.

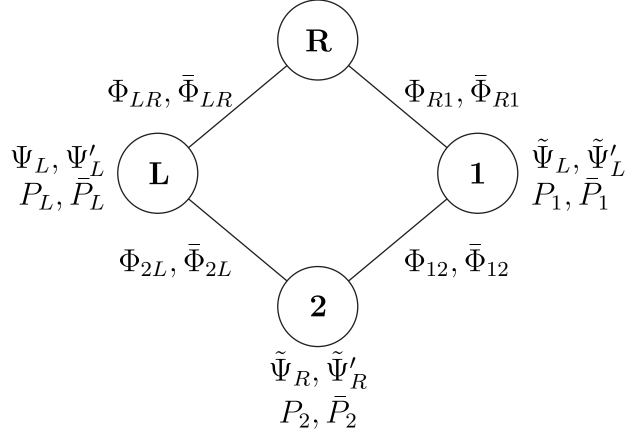

Figure 1: Quantum numbers of the matter and Higgs superfields under the four $SU(2)$ gauge groups in the model with matter in the bulk.

4.1 Matter in the Bulk

In our first model, the matter fields of the Standard Model arise as zero-modes of 5D fermions. One SM generation requires four 5D hypermultiplets, $(\Psi_L, \Psi'_L) \in (\mathbf{4}, \mathbf{2}, 1, 1, 1)$ and $(\Psi_R, \Psi'_R) \in (\bar{\mathbf{4}}, 1, \mathbf{2}, 1, 1)$, where in the brackets we list transformation properties under $SU(4) \times SU(2)_L \times SU(2)_R \times SU(2)_1 \times SU(2)_2$. Each 5D hypermultiplet Ψ consists of a Dirac fermion ψ and two complex scalars ϕ and ϕ^c. The boundary conditions on the scalars are,

$$
\begin{aligned}
\phi(x^\mu, x_5) &= +\phi(x^\mu, -x_5) = \mathcal{C}\, Z\, \phi(x^\mu, x_5 + 2\pi R), \\
\phi^c(x^\mu, x_5) &= -\phi^c(x^\mu, -x_5) = \mathcal{C}\, Z\, \phi^c(x^\mu, x_5 + 2\pi R),
\end{aligned} \tag{10}
$$

where $Z =$ diag$(1, 1, 1, -1)$, and $\mathcal{C} = \pm 1$ is the parity of the field Ψ. The fields $\psi_+ = \frac{1}{2}(1 + \gamma_5)\psi$ and $\psi_- = \frac{1}{2}(1 - \gamma_5)\psi$ have the same boundary conditions as ϕ and ϕ^c, respectively. The fields ϕ^c and ψ_- have no 4D zero modes. The fields ϕ and ψ_+ each have a zero mode which together form a 4D $N = 1$ chiral multiplet. The gauge charges of the zero modes of Ψ under $SU(3) \times U(1)_{B-L}$ depend on its parity: for $\mathcal{C} = +1$ the zero modes transform as a quark or an antiquark, while for $\mathcal{C} = -1$ they transform as a lepton or an antilepton. We assign $\mathcal{C} = +1$ to the fields Ψ_L and Ψ_R and $\mathcal{C} = -1$ to the fields Ψ'_L and Ψ'_R. As we already discussed in section 2, to be able to impose a discrete symmetry necessary for a correct prediction of the weak mixing angle we need to introduce four more 5D fields per SM generation: $(\tilde{\Psi}_L, \tilde{\Psi}'_L) \in (\mathbf{4}, 1, 1, \mathbf{2}, 1)$ and $(\tilde{\Psi}_R, \tilde{\Psi}'_R) \in (\mathbf{4}, 1, 1, 1, \mathbf{2})$. The zero modes of these fields form three additional "spectator generations", which however acquire masses at the scale M'_c as we will show below.

The Higgs sector of the model consists of:

- Eight 4D chiral superfields[4], Φ_{ij} and $\bar{\Phi}_{ij}$, where $(ij) = LR, R1, 12, 2L$. These fields are

[4]It is easy to construct phenomenologically viable models with fewer 4D bidoublet Higgses. We choose the

localized on the 3-1 boundary. They transform as bidoublets (or "link fields") under the corresponding $SU(2)$ groups, and are singlets of $SU(4)$.

- Eight 5D hypermultiplets, P_i and $\bar{P}_i$, $i = 1, 2, L, R$. These fields transform as $\mathbf{4}$ and $\bar{\mathbf{4}}$, respectively, under $SU(4)$, and as doublets under the corresponding $SU(2)$'s. The boundary conditions imposed on these fields are identical to (10) with $\mathcal{C} = -1$. For example, the zero modes of the P_R and $\bar{P}_R$ transform like 4D $N = 1$ chiral multiplets that have the quantum numbers of a right-handed lepton doublet and a right-handed antilepton doublet, respectively.

The matter and Higgs superfields of the model and their $SU(2)$ quantum numbers are conveniently summarized in Fig. 1.

We assume that the scalar components of Φ_{1R}, $\bar{\Phi}_{1R}$, Φ_{12} and $\bar{\Phi}_{12}$ acquire diagonal vacuum expectation values (vevs) at the scale M_c', breaking $SU(2)_R \times SU(2)_1 \times SU(2)_2$ down to the diagonal $SU(2)$ subgroup. (We do *not* break any of the $SU(2)$'s by boundary conditions in this model.) The vev of the scalar component of P_R at the same scale breaks the product of this $SU(2)$ and $U(1)_{B-L}$ down to the SM hypercharge group $U(1)_Y$. Finally, the SM electroweak symmetry breaking is achieved by the vevs of $\phi_u \equiv \Phi_{LR}$ and $\phi_d \equiv \bar{\Phi}_{LR}$, for which we assume the pattern[5]

$$\phi_u = \begin{pmatrix} v_u & 0 \\ 0 & 0 \end{pmatrix}, \qquad \phi_d = \begin{pmatrix} 0 & 0 \\ 0 & v_d \end{pmatrix}. \tag{11}$$

The rest of the Higgs fields do not acquire vevs.

Let us analyze the pattern of masses for the matter fields. To simplify the analysis, we assume that the Yukawa couplings satisfy a discrete Z_2 symmetry under which the "spectator generation" superfields ($\tilde{\Psi}_{L,R}$ and $\tilde{\Psi}'_{L,R}$) have a charge -1 and the rest of the fields are invariant. Then, the zero modes of the "spectator generations" get masses at the scale M_c' through the superpotential couplings $\tilde{\Psi}_L \Phi_{12} \tilde{\Psi}_R + \tilde{\Psi}'_L \Phi_{12} \tilde{\Psi}'_R$, while the three ordinary generations get masses at the weak scale through their couplings to ϕ_u and ϕ_d. In addition, the right-handed neutrino gets a Majorana mass of order 10 GeV through a non-renormalizable coupling $(\nu^c P_R)^2/M_s$.

The main challenge in the flavor sector of the model is to explain the smallness of neutrino Dirac masses: successful phenomenology requires $m_\nu^D \approx 10^4$ eV. We take the view that the neutrino Yukawa couplings vanish exactly due to a discrete symmetry. We postulate a symmetry $\phi_d \to e^{i\,\pi/2}\phi_d$, $\Psi'_L \to e^{i\,3\pi/2}$, with all the other fields being invariant. This symmetry allows superpotential Yukawa couplings $L\phi_d e^c + Q\phi_u u^c$, but forbids $L\phi_u \nu^c$ and $Q\phi_d d^c$. The down-type quarks acquire their masses as a result of spontaneous supersymmetry breaking on the 3-1 boundary at a scale of order M_c. The Kahler potential term $\int d^4\theta X^\dagger Q \phi_u^\dagger d^c/M_s$, where X is the supersymmetry breaking spurion, induces a down quark Yukawa coupling suppressed by $M_c/M_s \sim 0.01$

structure presented here because of its pleasing symmetry, see Fig. 1.

[5] This pattern of the electroweak symmetry breaking is chosen to enable us to impose a discrete symmetry protecting vanishing neutrino Yukawa couplings, see below.

– the correct order of magnitude to explain the hierarchy between the top and bottom masses. The corresponding term for the neutrino, $\int d^4\theta X^\dagger L\phi_d^\dagger \nu^c/M_s$, is still forbidden by the discrete symmetry. A neutrino Dirac mass of the right order of magnitude can be generated by known higher-dimensional mechanisms [17].

Apart from the boundary-localized Yukawa couplings, the model possesses a cyclic symmetry interchanging the four $SU(2)$ factors, implying $b'_R = 3b'_L$. Using the results of Section 3, we obtain

$$\sin^2\theta(M_Z) = 0.231 - 0.0031 \ln\left(\frac{M'_c}{200\text{ GeV}}\right) \pm 0.005, \tag{12}$$

where we have assumed a 2% uncertainty on the prediction as explained at the end of Section 3. Our model contains the exotic gauge bosons W'_{LR} and Z'_{LR} with masses of order M'_c; experimental limits on such bosons in the canonical left-right symmetric model [5] require that their mass be higher than about 800 GeV. In our model, the limits are somewhat higher due to a stronger coupling of the $B-L$ component of Z'_{LR}. However, the uncertainty in the prediction (12) is sufficiently high to allow values of M'_c as high as 5 TeV at 2σ level, so the model is phenomenologically consistent. On the other hand, the non-supersymmetric version of the same model does not look viable, since it requires $M'_c \lesssim 650$ GeV to reproduce the experimentally measured value of the weak mixing angle within 2σ.

There are new sources of flavor-changing neutral currents in our model. The tree-level exchanges of the additional neutral Higgs bosons in ϕ_u and ϕ_d contribute to $K\bar{K}$ and $D\bar{D}$ mass differences. Suppressing this contribution requires that the mixing between the third and the first two generations in the right-handed sector be about as small as in the left-handed sector. In additon, there is a contribution from the box diagrams involving W_R gauge bosons. However, the small mixings of the third generation and the large masses of the W_R bosons ensure that this effect is harmless.

4.2 Matter on the Boundary

We now consider the possibility that both the breaking of $SU(4)$ to $SU(3)\times U(1)_{B-L}$ and the breaking of $SU(2)_R\times SU(2)_1\times SU(2)_2$ to $U(1)_R\times U(1)_1\times U(1)_2$ are accomplished using boundary conditions. For each $SU(2)$ field the appropriate pattern of breaking is obtained by demanding

$$\begin{aligned} A_i^\mu(x^\mu, x_5) &= +A_i^\mu(x^\mu, -x_5) = T_i\, A_i^\mu(x^\mu, x_5 + 2\pi R)\, T_i^{-1}, \\ A_i^5(x^\mu, x_5) &= -A_i^5(x^\mu, -x_5) = T_i\, A_i^5(x^\mu, x_5 + 2\pi R)\, T_i^{-1}, \end{aligned} \tag{13}$$

where $\mu = 1\ldots 4$ and the index i denotes the $SU(2)$ gauge group and runs over $L, R, 1$ and 2. We choose $T_L = \text{diag}(+,+)$ for $SU(2)_L$ reflecting the fact that it is not broken by boundary conditions; for the other $SU(2)$ groups, we choose $T_R = T_1 = T_2 = \text{diag}(+,-)$. The quark and lepton fields are localized on the boundary where both the $SU(4)$ and the $SU(2)^3$ symmetries are broken. The cyclic exchange symmetry between the $SU(2)$ groups is also broken at this point. We assume that the quark and lepton fields transform as three generations of $\Psi_L \in (\mathbf{4}, \mathbf{2}, 1, 1, 1)$ and $\Psi_R \in (\bar{\mathbf{4}}, 1, \mathbf{2}^*, 1, 1)$, where for notational simplicity we show the transformation properties under $SU(4) \times SU(2)_L \times SU(2)_R \times SU(2)_1 \times SU(2)_2$. (The transformation properties under the group that remains unbroken on the boundary, $[SU(3) \times U(1)_{B-L}] \times SU(2)_L \times U(1)_R \times U(1)_1 \times U(1)_2$, can be easily obtained from these.) No "spectator" generations are required in this model.

The model we consider is supersymmetric. The additional fields of the higher dimensional gauge multiplet transform as in the equation above, with λ_{1+} transforming like A_μ and λ_{2+} and σ transforming like A_5. As before the fields A_μ and λ_{1+} have zero modes which combine to form a 4D $N = 1$ gauge multiplet. The matter fields on the boundary become 4D $N = 1$ chiral superfields.

We now consider the Higgs sector of the theory. The Higgs fields are assumed to live in the bulk of the space. We wish to break $U(1)_R \times U(1)_1 \times U(1)_2$ to the diagonal $U(1)_R$. We therefore introduce pairs of Higgs hypermultiplets in the bulk which we denote by Φ_{ij} and $\bar{\Phi}_{ij}$, where $(ij) = LR, R1, 12, 2L$. These behave like link fields, transforming as bidoublets under $SU(2)_i \times SU(2)_j$ but as singlets under all other gauge groups. For example Φ_{LR} and $\bar{\Phi}_{LR}$ transform as $(\mathbf{2}^*, \mathbf{2})$ and $(\mathbf{2}, \mathbf{2}^*)$ respectively under $SU(2)_L \times SU(2)_R$, but as singlets under the remaining gauge groups. Each hypermultiplet Φ_{ij} consists of a Dirac fermion ψ_{ij} and two complex scalars ϕ_{ij} and ϕ^c_{ij}. The transformation properties of the scalars under the orbifold are given by

$$\begin{aligned} \phi_{ij}(x^\mu, x_5) &= +\phi_{ij}(x^\mu, -x_5) = T_i\, \phi_{ij}(x^\mu, x_5 + 2\pi R)\, T_j^{-1}, \\ \phi^c_{ij}(x^\mu, x_5) &= -\phi^c_{ij}(x^\mu, -x_5) = T_i\, \phi^c_{ij}(x^\mu, x_5 + 2\pi R)\, T_j^{-1}. \end{aligned} \tag{14}$$

The fermions $\psi_{ij,+}$ transform exactly like the ϕ_{ij}, while the fermions $\psi_{ij,-}$ transform like ϕ^c_{ij}. The fields ϕ_{ij} and $\psi_{ij,+}$ have zero modes which combine to form 4D $N = 1$ chiral multiplets. The other fields have no zero modes. Similarly each hypermultiplet $\bar{\Phi}_{ij}$ consists of a Dirac fermion $\bar{\psi}_{ij}$ and two complex scalars $\bar{\phi}_{ij}$ and $\bar{\phi}^c_{ij}$. While $\bar{\phi}_{ij}$ and $\bar{\psi}_{ij,+}$ transform exactly like the ϕ_{ij} under the orbifold and have zero modes, the remaining fields transform like ϕ^c_{ij} and have no zero modes. The zero modes of $\bar{\Phi}_{ij}$ form a 4D $N = 1$ chiral multiplet that is vector-like with respect to the zero modes of Φ_{ij}.

It is possible to write superpotential terms for the even components of the Φ and $\bar{\Phi}$ fields on the symmetry breaking boundary. These can be used to generate a potential for the even components of $[\Phi_{R1}, \bar{\Phi}_{R1}]$ and $[\Phi_{12}, \bar{\Phi}_{12}]$ that breaks $U(1)_R \times U(1)_1 \times U(1)_2$ down to the diagonal $U(1)_R$. The even components of Φ_{LR} and $\bar{\Phi}_{LR}$ correspond to the up type Higgs and down type Higgs chiral multiplets of the Minimal Supersymmetric Standard Model (MSSM).

There remains the breaking of $U(1)_R \times U(1)_{B-L}$ down to $U(1)_Y$. For this purpose we introduce pairs of hypermultiplets in the bulk which we denote by P_i and $\bar{P}_i$. P_i and $\bar{P}_i$ transform as $\mathbf{4}$ and $\bar{\mathbf{4}}$ under $SU(4)$ respectively, as doublets under $SU(2)_i$ and as singlets under the other $SU(2)$ groups.

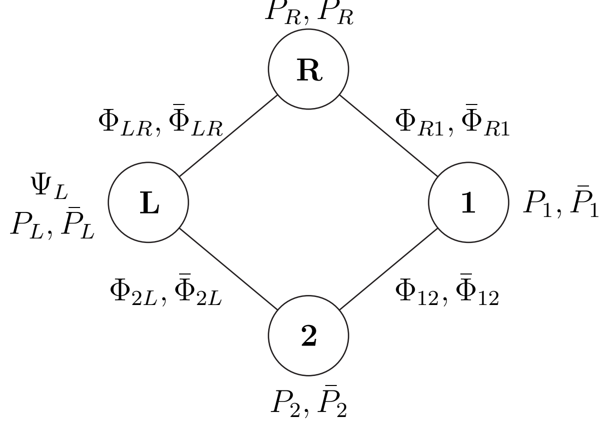

Figure 2: Quantum numbers of the matter and Higgs superfields under the four $SU(2)$ gauge groups in the model with matter on the boundary.

Each P_i consists of two complex scalars p_i and p_i^c, and a Dirac fermion ψ_{Pi}. The transformation properties of the scalars under the orbifold are,

$$\begin{aligned} p_i(x^\mu, x_5) &= +p_i(x^\mu, -x_5) = -ZT_i\, p_i(x^\mu, x_5 + 2\pi R), \\ p_i^c(x^\mu, x_5) &= -p_i^c(x^\mu, -x_5) = -ZT_i\, p_i^c(x^\mu, x_5 + 2\pi R). \end{aligned} \tag{15}$$

The fermions $\psi_{Pi,+}$ transform like p_i while the fermions $\psi_{Pi,-}$ transform like p_i^c. While $\psi_{Pi,+}$ and p_i have zero modes which combine to form 4D $N = 1$ chiral multiplets, the other fields have no zero modes. Similarly each of the hypermultiplets $\bar{P}_i$ consists of two complex scalars $\bar{p}_i$ and $\bar{p}_i^c$, and a Dirac fermion $\bar{\psi}_{Pi}$. While $\bar{p}_i$ and $\bar{\psi}_{Pi,+}$ share the same orbifold transformation properties as p_i and have zero modes which combine to form 4D $N = 1$ chiral multiplets, the other fields have no zero modes. The zero modes of P_i have the gauge quantum numbers of a neutrino of $SU(2)_i$ and a down quark of $SU(2)_i$ and $SU(3)$. The zero modes of $\bar{P}_i$ are vectorlike with respect to those of P_i. Then it is possible to write a potential for the zero modes of P_R and $\bar{P}_R$ (which have the quantum numbers of a right handed neutrino and a right handed anti-neutrino) on the boundary which breaks $U(1)_R \times U(1)_{B-L}$ down to $U(1)_Y$.

The Higgs and matter fields of the model and their $SU(2)$ quantum numbers are summarized in Figure 2. It is straightforward to generate fermion masses from Yukawa couplings between the Higgs fields and the matter fields on the symmetry breaking boundary. It is also not difficult to find a discrete symmetry which adequately suppresses the Dirac mass of the neutrino since there are no longer any mass relations from the larger gauge symmetry in the bulk.

From Eq. (7) we can obtain an expression for the compactification scale M_c. For concreteness we assume that supersymmetry is broken at the scale M_c' and therefore use the Standard Model expressions for b_Y, b_2 and b_3 when running below the compactification scale. Using the fact that bulk hypermultiplets do not contribute to any of the b's we find that $b_R' - 3b_2' = -12$. Then the expression for the weak mixing angle becomes

$$\sin^2\theta(M_Z) = 0.231 - 0.0065 \ln\left(\frac{M'_c}{1.74\ \text{TeV}}\right) \pm 0.005. \tag{16}$$

Even without invoking the uncertainties, the values of M'_c required to reproduce the experimentally measured value of $\sin^2\theta$ are sufficiently high to evade experimental constraints. The central value of the compactification scale is given by $M_c = \pi M'_c = 5.5$ TeV.

5 Conclusions

The first theory to unify quarks with leptons was based on the group $SU(4) \times SU(2)_L \times SU(2)_R$, and provided an elegant understanding of the fermion gauge quantum numbers [14]. We have shown that an extension to $SU(4) \times SU(2)^4$ allows this Pati-Salam structure to be realized at the TeV scale, leading to a tree-level prediction for the weak mixing angle, $\sin^2\theta = 0.239$, which is remarkably close to data. Such a TeV scale unification can be combined with extra-dimensional solutions of the hierarchy problem, by adding very large dimensions [12], or by adding an extra dimension with a warp factor [13] and placing the extended Pati-Salam sector on the TeV brane.

We do not view the extension from $SU(2)^2$ to $SU(2)^4$ as a major complication of the theory. In both cases a discrete symmetry between the $SU(2)$ factors is necessary to obtain a prediction for the weak mixing angle, and this symmetry must be broken, protecting only $SU(2)_L$ to lower energies. Various interpretations of the four $SU(2)$ factors are possible. For example, in the model of section 4.1 the Pati-Salam structure is simply repeated: $SU(2)_L \times SU(2)_R \times SU(2)'_L \times SU(2)'_R$. Another possibility, not pursued in this paper, is to have a different $SU(2)_{R_a}$ for each generation a: $SU(2)_L \times \prod_a SU(2)_{R_a}$. The usual $SU(2)_R$ is then just the diagonal sum of the $SU(2)_{R_a}$, leading to the desired prediction for the weak mixing angle.

We have realized the $SU(4) \times SU(2)^4$ symmetry in 5 dimensions, with boundary conditions in the compact fifth dimension breaking $SU(4) \to SU(3) \times U(1)_{B-L}$. This facilitates imposing the discrete symmetry among the $SU(2)$ factors, removes the powerful constraint from $K_L \to \mu e$ on the masses of the charged $SU(4)$ gauge bosons, leads to Yukawa couplings which are not $SU(4)$ symmetric and allows baryon number to be a symmetry of the theory. Providing the five dimensional effective theory is valid up to a scale where the gauge couplings approach strong coupling, the prediction for the weak mixing angle is under control [2, 7], and our result, including radiative corrections and uncertainties, is shown in (8) and (9) for this class of theories.

There are many possibilities for the breaking pattern of $SU(2)^3 \times U(1)_{B-L} \to U(1)_Y$, leading to model dependence in the weak mixing angle prediction and in the signatures at future collider experiments. We have given two explicit supersymmetric models; one having the three $SU(2)$s broken by boundary conditions, and the other having the symmetry breaking entirely from the

Higgs mechanism. In both cases, the weak mixing angle is successfully predicted, although the central values for the compactification scale differ: 5.5 TeV for boundary condition breaking, and 600 GeV for Higgs breaking. However, there is an order of magnitude uncertainty in these values of the compactification scale arising from the uncertainty in the prediction of the weak mixing angle. While theories of $SU(4) \times SU(2)^4$ unification predict many new phenomena at the TeV scale, including heavy W' and Z' bosons and KK modes for all gauge bosons, there is a significant uncertainty in the precise energy threshold for this new physics. The reach of the Tevatron and the LHC for the neutral Z' gauge boson will be high: its $B - L$ component couples with a QCD strength gauge coupling, it is singly produced in $q\bar{q}$ collisions (or pair-produced in gg collisions), and decays readily to charged quark or lepton pairs. The quark and lepton branching ratios will reveal that this Z' boson is coupled to $\frac{1}{2}(B - L) - T_{3R}$, while its production cross section will indicate that $SU(4)$ unification occurs at a low scale.

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